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Neutrino Masses and the Voloshin-Vysotsky-Okun Solution to the Solar Neutrino Problem

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Abstract

We analyse the problem of reconciling small neutrino masses with the large electron neutrino magnetic moment $\mu \sim (0.3 - 1) \times 10^{10}\mu_B$ required for the Voloshin-Vysotski-Okun solution to the solar neutrino puzzle. We point out that the relations of plausible see-saw arguments can be upset if neutrino mass to lepton mass ratios run with energy, but can be restored to reasonable values at high energy. We present a solution in which neutrino masses are naturally close to their laboratory bounds. In particular $2MeV \lesssim m(\nu_\tau) \lesssim 50MeV$ and the branching ratio $\mu \to e\gamma$ must be finite.

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1.) One of the unsolved problems of the standard electroweak model[1] is the neutrino puzzle. Experiments thus far allow for a massless neutrino, although it may well be that neutrinos are massive but very light as compared to the other fermions. Understanding the smallness of the neutrino mass requires treating this particle on a different footing from other fermions.

If right-handed neutrinos are added to the standard model, the currently popular see-saw mechanism[2] can generally be invoked to give any small Dirac mass $m(\nu)$ at the expense of large Majorana masses. The logic involves $SU(2)$ singlet interactions of the right-handed neutrinos which could be associated with a large scale M. An additional price one pays is that in some cases this high scale m may not be within the reach of present or future experiments. For example, Harari and Nir[3] quote a value of $M \gtrsim 50PeV$, which is quite discouraging. Thus the see-saw mechanism, while attractively general,requires additional elements to be testable. Therefore it is useful to continue seeking explanations for the smallness of neutrino masses. A second aspect of the neutrino puzzle is the well-known deficit in the observed capture rate of solar electron neutrinos[4] as compared to the calculations of the standard model. Even more significant is the observation of synchronized periodic time dependence of the neutrino flux which is anti-correlated with the sunspot number[5]. Large numbers of sunspots occur with large values of the solar magnetic field, so that the anti-correlation can be explained if one assumes a large neutrino magnetic moment. Voloshyn, Vysotski and Okun[6] (VVO) argue that a neutrino magnetic moment $\mu_\nu \sim 0.3-1.0\times 10^{10}\mu_B$, where $\mu_B = e/2m_e c$ is the Bohr magneton, can explain the data. The mechanism hinges on conversion of left-handed electron neutrinos to right-handed neutrinos through the helicity-flipping moment coupling, the process depending on the depth and magnitude of the solar magnetic field. The magnetic moment required is seven to nine orders of magnitude larger than standard model estimates.[7] Even so, uncertainties in the solar magnetic field, as well as more subtle questions of the importance of coherent (MSW) effects[8] and normal mixing make the VVO implementation of the idea somewhat tentative. A reasonable window to work with in considering VVO is $\mu_\nu \sim 10^{-11} - 10^{-10}\mu_B$, keeping in mind that flavor off-diagonal neutrino transitions could be important and the firm laboratory bound[9] $\mu_\nu < 2 \times 10^{-10}\mu_B$.

2.) In this paper we investigate a model that is capable of producing a large neutrino magnetic moment. We consider reconciling the largeness of the magnetic moment with the smallness of neutrino masses. This is a serious problem since large mass corrections tend to be generated by the same type of diagrams that give large magnetic moments. However, since the mass corrections are logarithmically divergent, they are impossible to interpret until set into a renormalization group context. We will see that the predictive power of the see-saw arguments is thus enhanced.

Conventional wisdom assumes that neutrino masses, when finally observed, will obey a hierarchy $m(\nu_\tau) > m(\nu_\mu) > m(\nu_e)$. Harari and Nir[3,10] have argued that in any "resonable see-saw" picture one can expect to find the more detailed relation

$$\frac{m(\nu_i)}{m(\nu_j)} \cong \left(\frac{m_i}{m_j}\right)^P \quad 1 \lesssim P \lesssim 2 \tag{1}$$

where m_i is the mass of the charged leptons. For $P = 1$, this relation predicts the tau neutrino to have a mass vastly smaller than current direct measurements. Taking $m(\nu_e) \lesssim 20eV$, we find $m(\nu_\tau) \lesssim 70keV$, about 1000 times smaller than the ARGUS bound[11] $m(\nu_\tau) \lesssim 50MeV$. A similarly unsatisfactory situation occurs for $m(\nu_\mu) \lesssim 270keV$, the current laboratory bound.

The relation (1), however, is not specific about the measurement scale of the mass parameters. That is, such relations could be upset by renormalization effects, if the neutrinos and charged leptons run with energy at different rates.

Let us assume the reasonable hierarchy (1) applies at a high-energy scale Λ where symmetry breaking and mass generation takes place. We call this a "bare" resonable see-saw, as opposed to the renormalized case. A crude estimate of the change in masses $\Delta m(\nu_k)$ as measured at a laboratory scale μ^2 is given by

$$\Delta m(\nu_k) \cong C_k \ln(\Lambda^2/\mu^2) \tag{2}$$

where C_k is a calculable coefficient, related to the anomalous dimension of the running mass. We know (2) is not the same as the solutions of exact renormalization group equations, which include running coupling effects. However it is a reasonable estimate, which we could expect to work within a factor of 2 or 3 or so, just like the reasonable relation (1). (Note that quark masses even within the same doublet and at the same scale can differ by an order of magnitude.)

If (2) is used, one considers whether there can occur much <u>different</u> constants

C_k for the neutrinos as compared to charged leptons. We will show that this indeed happens in the model we study. One possible and attractive picture we obtain is that the electron neutrino mass runs the fastest, departing from the magic ratio (1) to an anomalously small value at low energy. This can occur precisely because the large magnetic moment of the electron neutrino assumes large radiative corrections. Moreover, the analysis is restrictive enough to constrain the other neutrino masses, and we find that they do not have to be unreachably small when measured at laboratory scales.

The large mass values require short neutrino lifetimes to satisfy cosmological bounds. The short lifetimes, however, occur naturally with the large magnetic moments in the model via transitions $\nu_j \to \nu_i + \gamma$. Thus we find several predictions from the overall consistency of the picture.

3.) In the standard model, $\mu_\nu \sim 10^{-19}\mu_B(m_\nu/eV)$ is small[7,12] because of a GIM suppression. However, in general μ_ν does not have any direct relation to m_ν. The magnetic moment, as a dimension-five interaction, does probe radiative corrections rather uniquely. Fukugita and Yanagida[13] show that μ_ν of the order needed by VVO is indeed possible through one-loop radiative corrections in a simple extension of the GWS model.

The model[13–15] introduces one new charged scalar particle η, an $SU(2)$ singlet carrying lepton number 2. Individual lepton numbers L_j for $j = 1,2,3$ for e, μ, τ are not necessarily conserved, but total lepton number is conserved. The most general lepton interaction with η, renormalizable by power counting, is

$$\mathcal{L}_{\text{int}} = g_{ij}\bar{L}_L^{c,i} L_L^j \eta^+ + f_{ij}\bar{\nu}_R^{c,i} e_R^j \eta^+ \quad , \tag{3}$$

where $L = (\nu, e^-)_L$ is the usual $SU(2)$ doublet, i, j are flavor indices and c denotes charge-conjugation. The couplings are limited to $g_{ij} = -g_{ji}$ and $f_{ij} = -f_{ji}$ by $SU(2)_L$ and $SU(2)_R$ symmetry, respectively, the latter imposed for simplicity. Parity conservation in the η interaction can be imposed to give $f_{ij} = g_{ij}$. The authors of Ref. (13) argue that the η-scalar might be observed in the mass range $25GeV < m_\eta < 1TeV$, the lower bound based on production searches.

The main constraint on (3) comes from the smallness of $\mu \to e\gamma$, which requires a small relative value for the couplings

$$f_{23}f_{31}^{\dagger} + g_{23}g_{31}^{\dagger} \to 0 \quad . \tag{4}$$

If we impose parity conservation[1] and real couplings we have $f_{ij} = g_{ij}$, implying from (4)

$$g_{23}g_{31} \to 0 \quad . \tag{5}$$

Dropping parity conservation generally makes all relations easier to satisfy. For this work we usually interpret (5) rather literally, although strictly speaking data only requires[16] the $\mu \to e\gamma$ branching ratio to be less than 2×10^{-10}. We obtain three cases, we discuss in detail, (I) $g_{13} \to 0$; (II) $g_{13}, g_{23} \to 0$; (III) $g_{23} \to 0$.

The one-loop diagrams of Fig. 1 give rise to the neutrino magnetic moment. Generally there is a transition moment matrix[17] μ_{ij}, defined by $\mathcal{L} = \frac{1}{2}\mu_{ij}\bar{\nu}_R^i \sigma^{\mu\nu} F_{\mu\nu} \nu_L^j$, that can be calculated to give[13,15,17]

$$\begin{aligned} \mu_{ij} &= \frac{e}{32\pi^2} \sum_l \left(g_{il} f_{lj}^{+} + f_{il} g_{lj}^{+} \right) \frac{m_l}{m_\eta^2} \left[ln(m_\eta^2/m_l^2) - 1 \right] \quad , \\ &= \frac{e}{16\pi^2} \sum_l g_{il} g_{lj} \frac{m_l}{m_\eta^2} \left[ln(m_\eta^2/m_l^2) - 1 \right] \quad , \end{aligned} \tag{6}$$

where m_l is the mass of the l^{th} charged lepton inside the loop. Suppressing the logarithm and numerical factors in (6), the VVO transitions of electron neutrinos in the sun could include the following:

$$\begin{aligned} \nu_1 + \gamma \to \nu_1 \quad &; \quad g_{13}g_{31}m_3 + g_{12}g_{22}m_2 \\ \nu_1 + \gamma \to \nu_2 \quad &; \quad g_{13}g_{32}m_3 \\ \nu_1 + \gamma \to \nu_3 \quad &; \quad g_{12}g_{23}m_2 \quad . \end{aligned} \tag{7}$$

Thus a sizeable magnetic moment in the three cases and the $\mu \to e\gamma$ suppression

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[1] In some cases, (*e.g.* g_{12} quote large) it cannot be assumed that $f_{ij} = g_{ij}$ is automatically consistent with all experimental constraints. In particular, the interference of conventional $V - A$ terms in μ decay with g_{12} shows that g_{12} cannot be too large[15]. The more general relation $g_{ij} = \alpha f_{ij}$ is consistent with all constraints. Since all mass corrections and magnetic moment scale like fg, nothing in our analysis depends on the parameter α and it can be safely absorbed in the values here quoted for g^2

will fix a combination of the couplings going like

$$
\begin{array}{lll}
\text{(I) } g_{23} \to 0: & g_{13}g_{31}m_3 + g_{12}g_{21}m_2\ ; & \text{transitions } \nu_1 \leftrightarrow \nu_1 \text{ only } . \\
\text{(II) } g_{13}, g_{23} \to 0: & g_{12}g_{21}m_2\ ; & \text{transitions } \nu_1 \leftrightarrow \nu_1 \text{ only } . \\
\text{(III) } g_{13} \to 0: & g_{12}g_{23}m_2, g_{12}g_{21}m_2\ ; & \text{transitions } \nu_1 \leftrightarrow \nu_1, \nu_1 \leftrightarrow \nu_3\ .
\end{array}
$$

Note that only in case III is there the possibility of off-diagonal transitions to deal with. To obtain a least restrictive condition for VVO to work, we extend the naive moment condition $\mu_\nu \cong (0.3 - 1) \times 10^{-10}\mu_B$ to allow the full probability of $\nu_1 + \gamma$ to be the needed order of magnitude:

$$\mu_{11}^2 + \mu_{12}^2 + \mu_{13}^2 \cong 10^{-21}\mu_B^2 \quad . \tag{8}$$

We have chosen the lower bound to impose the weakest limits. The bulk of our analysis does not depend much on the details of this generalization, as it will become clear below that allowed transitions are almost purely diagonal (μ_{11}). Thus either the naive VVO condition or the generalized one (8) can be used self-consistently.

Let us also study the one-loop neutrino mass correction matrix Δm_{ij}, as calculated from Fig. 2. We have

$$\Delta m_{ij} = \begin{pmatrix} 2(f_{12}g_{21}m_2 + f_{13}g_{31}m_3) & (f_{13}g_{32} + g_{13}f_{32})m_3 & (f_{12}g_{23} + g_{12}f_{23})m_2 \\ (f_{13}g_{32} + g_{13}f_{32})m_3 & 2(f_{23}g_{32}m_3 + f_{21}g_{12}m_1) & (f_{21}g_{13} + g_{21}f_{13})m_1 \\ (f_{12}g_{23} + g_{12}f_{23})m_2 & (f_{21}g_{13} + g_{21}f_{13})m_1 & 2(f_{31}g_{13}m_1 + f_{32}g_{23}m_2) \end{pmatrix} \times \mathrm{I} \tag{9}$$

where[18] I is the logarithmically divergent integral

$$\begin{aligned} \mathrm{I}\left(p^2, m_\eta^2, \Lambda^2; m_l^2\right) &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{\left(k^2 - m_\eta^2\right)\left((p-k)^2 - m_l^2\right)} \\ &= \frac{1}{16\pi^2} ln\left(\Lambda^2/m_\eta^2\right) + \text{ finite} \end{aligned} \tag{10}$$

and in the limit m_η^2 and $\Lambda^2 \gg m_l^2$. Here Λ^2 is an ultraviolet cutoff; a value of Λ^2 can be chosen to parameterize the finite parts of mass counter terms. Note that the charged lepton masses m_l in (9) tend to be large compared to neutrino masses, posing the problem of interpretation we referred to in Section 2.

4.) We now consider starting with typical laboratory mass values for neutrinos and renormalizing up in scale, to predict mass ratios at high energy. We understand that a more conventional procedure might begin at high energy and renormalize down, but we wish to make use of data measured at low energies. We first note that from neutrino oscillation experiments[19] the mixing is either nearly or $90°(\sin^2 2\phi \ll 1)$ between weak eigenstates to form mass eigenstates, barring the case of very tiny Δm^2. Since in the mass matrix (9) the logarithm is an overall factor, the basis which diagonalizes the correction Δm_{ij} does not change with scale, to leading order, so if we diagonalize at low energy the basis stays good at high energy. However, only for certain ranges of the coupling constants g_{12}, g_{23}, g_{13} is the weak eigenstate a mass correction eigenstate, so we may restrict attention to such ranges. For a diagonalized matrix, the crude multiplicative scaling (2) is reasonable, as the change in each matrix element is decoupled from the others.

Imposing parity conservation in case I ($g_{23} \to 0$) we have

$$\Delta m_{ij} = \frac{1}{16\pi^2}\begin{pmatrix} -2(g_{12}^2 m_2 + g_{13}^2 m_3) & 0 & 0 \\ 0 & -2g_{12}^2 m_1 & -2g_{12}g_{13}m_1 \\ 0 & -2g_{12}g_{13}m_1 & -2g_{13}^2 m_1 \end{pmatrix} \tag{11}$$

which leads us to the following set of eigenvalues and eigenstates:

$$\begin{aligned} \Delta m_1' &= -2(g_{12}^2 m_2 + g_{13}^2 m_3)/16\pi^2 \,;\, |1'>=|\nu_1> \;; \\ \Delta m_2' &= 0 \,;\, |2'>=\cos\vartheta|\nu_2> - \sin\vartheta|\nu_3> \;; \\ \Delta m_3' &= -2(g_{12}^2 + g_{13}^2)m_1/16\pi^2 \,;\, |3'>=\sin\vartheta|\nu_2> + \cos\vartheta|\nu_3> \;, \end{aligned} \tag{12}$$

where we have made

$$\cos\vartheta = \frac{g_{13}}{\sqrt{g_{12}^2+g_{13}^2}} \;;\; \sin\vartheta = \frac{g_{12}}{\sqrt{g_{12}^2+g_{13}^2}} \;.$$

We thus see that the $\mu-$ and $\tau-$ neutrinos combine to form an approximately invariant eigenstate and a state with a correction proportional to the electron mass. The other eigenstate coincides with the electron neutrino and has a correction to $m(\nu_e)$ set by the muon and tau masses.

For case II ($g_{13}, g_{23} \to 0$) the mass matrix can be read off from (11). The matrix is diagonal in the flavor basis with $\Delta m(\nu_3) \approx 0$ and $\Delta m(\nu_1), \Delta m(\nu_2)$ going like m_2 and m_1, respectively.

For case III ($g_{13} \to 0$) the matrix (9) reads

$$\Delta m_{ij} = \frac{1}{16\pi^2} \begin{pmatrix} -2g_{12}^2 m_2 & 0 & -2g_{12}g_{23}m_2 \\ 0 & -2(g_{12}^2 m_1 + g_{23}^2 m_3) & 0 \\ -2g_{12}g_{23}m_2 & 0 & -2g_{23}^2 m_2 \end{pmatrix} \quad (13)$$

with eigenvalues and eigenstates

$$\begin{aligned} \Delta m_1' &= 0 \ ; \ |1'> = \cos\phi|\nu_1> - \sin\phi|\nu_3> \ ; \\ \Delta m_2' &= -2\left(g_{12}^2 m_1 + g_{23}^2 m_3\right)/16\pi^2 \ ; \ |2'> = |\nu_2> \ ; \\ \Delta m_3' &= -2\left(g_{12}^2 + g_{23}^2\right) m_2/16\pi^2 \ ; \ |3'> = \sin\phi|\nu_1> + \cos\phi|\nu_3> \ , \end{aligned} \quad (14)$$

where

$$\cos\phi = \frac{g_{23}}{\sqrt{g_{12}^2 + g_{23}^2}} \ ; \ \sin\phi = \frac{g_{12}}{\sqrt{g_{12}^2 + g_{23}^2}} \quad . \quad (15)$$

We thus find that the simultaneous requirements of parity conservation and $\mu \to e\gamma$ suppression leads to one invariant and two corrected masses. In case II, where $g_{13}, g_{23} \sim 0$, the mass matrix is diagonal and the neutrino mass corrections do not parallel the corresponding charged lepton masses. In case III the invariant neutrino is a combination of the electron and tau neutrino, while the muon neutrino changes its mass with no mixing.

5.) A case giving a large correction to $m(\nu_e)$ is case III, $g_{12} \gg g_{23}$, so $\sin\phi \to 1$. Let us show how this example can reconcile the VVO moment and the reasonable see-saw relation.

(1). For numerical estimates we take a minimum value of ln $(\Lambda^2/m_\eta^2) \to \mathbf{1}, g_{23} \to 0$ and $g_{12} \gtrsim 1/19$, the last from the VVO relation (8). A detailed discussion of coupling limits is given in Sec. 7.

For these couplings the state $|3'> \simeq |\nu_e>$ from (14,15) so $\Delta m_3' \cong \Delta m(\nu_e) \cong -3.8keV$ with the above values. The other corrections are much smaller, $\Delta m(\nu_\mu) \cong -18eV$ and $\Delta m(\nu_\mu) \cong 0$. Corrections of this order are negligible for the charged leptons. However, we see that the $m(\nu_e)/m_e$ ratio is receiving a large change: at high energy it could be as much as $(3.8keV)/511keV = 7.4 \times 10^{-3}$. The situation for the mass ratios in this case at high energy becomes

$$\frac{m(\nu_e)}{m_e} : \frac{m(\nu_\mu)}{m_\mu} : \frac{m(\nu_\tau)}{m_\tau} = \begin{array}{c} 7.4 : 2.5 : 28 \\ \times 10^{-3} \end{array} \tag{16}$$

using the renormalized laboratory bounds. Note these values are quite acceptable, although $m(\nu_\tau)/m_\tau$ is large. Of course the bounds are upper limits, so $m(\nu_\tau)/m_\tau$ can be smaller and still consistent: with a value 4-10 times smaller than the laboratory bound this ratio would come closer to the other two. That would occur if the low energy $m(\nu_\tau)$ were found to be $m(\nu_\tau) \simeq 12.5 - 5MeV$, a value that could be measured in the not-too-distant future.

Recalling that the "bare" $P = 1$ see-saw required $m(\nu_\tau)$ to be 1000 times (or more) smaller than the laboratory bound, (16) is a much more optimistic prediction. Moreover, the low value of $m(\nu_\mu)/m_\mu$ indicates that if the ratios are to be believed, then $m(\nu_\mu)$ should not be too far from the current bounds. However, if one found that $m(\nu_\mu)$ were very much smaller that its current bound, then it would rule out our proposal, since $m(\nu_e)$ is at its lowest high energy value already. While such relations cannot be conclusive in the absence of real measurements of the neutrino masses, (16) demonstrates that the ratios can be renormalized to satisfactory values. We conclude that one would not throw out the model on the basis of mass corrections; indeed, the results go in the direction of improving the situation.

The reader can check that the interesting case we have just studied is not the only one that can be consistent with data. For example, case III $g_{23} \gg g_{12}(\sin\phi \to 0)$ can be made consistent, at least up to this stage. However, such a case has little to say about renormalizing the reasonable see-saw. With $m(\nu_e)$ invariant, one is stuck with $m(\nu_\tau)/m_\tau \leq 4 \times 10^{-5}$ in the $P = 1$ case. Consequently, we would have nothing interesting to say about $m(\nu_\tau)$. Similarly, the $P = 2$ cases are generally not so interesting as the $P = 1$ limit, as one can always cook-up a neutrino mass that will satisfy the bounds, without being able to say much else. Leaving aside such situations, we now continue with the interesting case to study the neutrino lifetime.

6.) The arguments above have shown that it is plausible for the neutrino masses to have values close to their current laboratory limits. However the laboratory limits are generally not as stringent as the more model dependent cosmological limits that can be imposed. There is no way we can come close to the stable neutrino limit[20]

$\sum_i m(\nu_i) \lesssim 60\ eV$ for all the neutrinos. This leaves the case of unstable μ and τ neutrinos.

For unstable neutrinos we have the bound[20,10]

$$m^2(\nu_j)\tau_\nu \lesssim 2 \times 10^{20} eV^2\ sec \tag{17}$$

where τ_ν is the neutrino lifetime, assuming $\tau_\nu < 10^{17}\ sec$, the age of the universe. The radiative decay $\nu_j \to \nu_i + \gamma$ can be calculated in the model. Such decays[17] go through μ_{ji} the transition magnetic moments given by (6). The rate[17] $\Gamma_\nu = \frac{1}{\tau_\nu}$ is

$$\begin{aligned} \Gamma_\nu &= (\mu_{ji})^2 m(\nu_j)^3/8\pi; \\ \tau_\nu &= 1.9 \times 10^{19}\ sec\ \left(10^{-10}\mu_B/\mu_{ji}\right)^2 (eV/m(\nu_j))^3, \end{aligned} \tag{18}$$

in the limit $m(\nu_j)/m(\nu_i) \gg 1$. A convenient formula combining the bound and radiative lifetime is

$$m(\nu_j) \gtrsim \frac{1}{12}\left(\frac{10^{-10}\mu_B}{\mu_{ji}}\right)^2 eV. \tag{19}$$

In many models such a bound can be hard to manage, as the decay rate tends to be too slow. Let us see if the μ and τ neutrinos can satisfy (19) for our solution.

The following moments are needed:

$$\begin{aligned} \nu_\tau \to \nu_e + \gamma \ &\Rightarrow\ \mu_{31} \sim g_{32}g_{21}m_2/m_\eta^2 \\ \nu_\mu \to \nu_e + \gamma \ &\Rightarrow\ \mu_{21} \sim g_{23}g_{31}m_2/m_\eta^2 \end{aligned}$$

These decays depend on the couplings g_{23}, g_{31} that we suppressed earlier. Given $m(\nu_\tau) \gg m(\nu_\mu), g_{12} \gg g_{23}, g_{13}$, we need only check that μ_{21} is large enough to show whether both ν_τ and ν_μ pass the unstable bound.

A bound on the μ_{21} moment can be obtained from the $\mu \to e\gamma$ rate as follows. The rate $\Gamma(\mu \to e\gamma)$ is given by [13]

$$\Gamma(\mu \to e\gamma) = \frac{1}{8\pi} m_\mu^3 \left[\frac{e}{32\pi^2}\frac{m_\mu}{6m_\eta^2} g_{13}g_{32}\right]^2 \lesssim 10^{-4}/sec,$$

the second relation coming from the muon lifetime and branching ratio. We then obtain a limit

$$\frac{em_\mu}{16\pi^2 m_\eta^2} g_{13}g_{32} \leq 4.5 \times 10^{-14}\mu_B \tag{20}$$

which we can use to bound μ_{21}. Comparing (20) to (6) we must supply a factor $m_\tau/m_\mu \ell n(m_\eta^2/m_\tau^2 - 1) \geq 77$ to bound the neutrino moment μ_{21}. We get $\mu_{21} \lesssim 3.5 \times 10^{-12}\mu_B$ for any reasonable value of m_η. Finally, we check the lower bound on $m(\nu_\mu)$ using μ_{21} at its maximum value:

$$\begin{aligned} m(\nu_\mu) &\gtrsim \frac{1}{12}\left(\frac{10^{-10}\mu_B}{3.5 \times 10^{-12}\mu_B}\right)^2 eV = 68\ eV \\ \tau(\nu_\mu) &\simeq 1.07 \times 10^6\ \mathrm{s} \end{aligned} \tag{21}$$

Even using the present $\mu \to e\gamma$ branching ratio bound, the value $m(\nu_\mu) \simeq 250\ keV$ in our ratio relations (16) is easily consistent with (21). However, it is quite possible that the $\mu \to e\gamma$ transition moment is an order of magnitude or more smaller than the present upper limit. In that case, the muon neutrino mass bound (21) becomes $m(\nu_\mu) \gtrsim 6.8\ keV$ which is a much more restrictive situation. Of course, the couplings g_{13}, g_{23} implied by (20) are consistent with our diagonalization case III as long as we keep $g_{12} \gg g_{23} \gg g_{13}$. The product $g_{13}g_{23}$ fixed by (20) is small enough to keep this well satisfied.

We note that even if the upper bound on the branching ratio of $\mu \to e\gamma$ decay is improved by as much as 3 orders of magnitude, the cosmological bound from (21) on $m(\nu_\mu)$ can still be satisfied. Finally, if the decay $\mu \to e\gamma$ were absolutely forbidden, then this bound on the mass of $m(\nu_\mu)$ would fail. Thus, the overall consistency of the picture requires both a finite branching ratio $\mu \to e\gamma$ and a finite mass for $m(\nu_\mu)$.

7.) The feature of zero mass corrections at one–loop order is interesting and suggests we study the mass matrix from a different point of view. We might adopt the one–loop values literally, comparing the corrections with data, and hope to find a justification for dismissing running mass effects if such a comparison worked. Here we will show that such an interpretation is not consistent with data in any case: we present it for completeness.

It is possible to examine cases I–III systematically. In case I, we must have $g_{13} \gg g_{12}$ for small mixing and the solution of coupling constants is the same as in Ref. (13). The tau mass fixes the diagonal moment μ_{11}, and one can show $g_{13}^2 \cong 10^{-6} m_\eta^2/GeV^2$. Given a lower bound $m_\eta \gtrsim 25\ GeV, g_{13} \gtrsim 1/40$ follows. Then the value of the electron neutrino mass m_1' in (12) is larger than $|2g_{13}^2 m_3/16\pi^2| \approx 14\ keV$. Comparing this to the laboratory bounds $m(\nu_e) \lesssim 30\ eV$, this case fails the literal interpretation by two orders of magnitude.

For case II ($g_{13}, g_{23} \to 0$), the mass matrix is diagonal. We first impose the VVO magnetic moment constraints from (8). Note that in either case II or case III μ_{12} is small, while μ_{11} and μ_{13} go like m_2. We then obtain a numerical condition upon combining (8) with (6):

$$g_{12}\sqrt{g_{12}^2 + g_{23}^2} \cong \frac{4.5 \times 10^{-5}(m_\eta^2/GeV^2)}{\ell n[m_\eta^2/m_2^2] - 1}. \tag{22}$$

The relation above assumes the lowest VVO value $\mu \sim 0.3 \times 10^{-10}\mu_B$ as the least restrictive, and the numerical value given has incorporated the muon mass. We compare (22) with the mass eigenvalues and laboratory bounds

$$\begin{aligned} m_1' = 2g_{12}^2 m_2/16\pi^2 &\lesssim 30\ eV \\ m_2' = 2g_{12}^2 m_1/16\pi^2 &\lesssim 270\ keV \\ m_3' = 0 &\lesssim 50\ MeV. \end{aligned} \tag{23}$$

We need $g_{12} \lesssim 1/211$ to satisfy $m_1' \lesssim 30\ eV$. This in turn implies an upper bound $m_\eta \lesssim 1.5$ GeV using VVO (22). This is sufficient to rule out case II.

For case III ($g_{13} \to 0$) neutrino oscillation experiments are important. We see that mass differences in the $e - \tau$ sector are large (c.f. (14)) so we must be in a region of small mixing combination $\sin^2 2\phi \ll 1$ to be consistent with data. For this discussion, we interpret data as requiring

$$|\sin 2\phi| = \frac{|2g_{13}g_{23}|}{g_{12}^2 + g_{23}^2} \lesssim (3 \times 10^{-3})^{1/2} \tag{24}$$

This is an estimate, but reasonable since data absolutely prohibits $\sin^2 2\phi_{13}$ as large as 10^{-2} for anything but very tiny mass differences. The ratio $|g_{12}/g_{23}| = |\tan\phi| = t$ can be determined from (24), giving two possibilities

$$t^2 - 36.5t + 1 \gtrsim 0 \Rightarrow t \gtrsim 36.5 \text{ or } t \lesssim 1/36.5 \tag{24a}$$

For large mixing, (15) is at nearly ninety degrees. So, if $g_{12} \gg g_{23}$, we have $|\sin\phi| \ll 1$ and $|3'> \simeq |\nu_1>$ while $|1'> \simeq -|\nu_3>$. Thus, to an excellent approximation, the laboratory electron neutrino mass bound applies to m_3', so that

$$2g_{12}^2 m_2/16\pi^2 \lesssim 30\ eV \tag{25}$$

Furthermore, the VVO condition (22) simplifies, reading

$$g_{12}^2 \simeq \frac{4.5\times 10^{-5} m_\eta^2/GeV^2}{\ell n(m_\eta^2/m_2^2)-1}. \tag{26}$$

Requiring $m_\eta > 25$ GeV, (26) gives $g_{12}^2 \gtrsim 2.8\times 10^{-3}$, a value too large to satisfy inequality (25).

For small t, the mass and flavor eigenstates nearly coincide and g_{23} is the large coupling. In this case, condition (22) becomes

$$g_{23} \simeq \frac{1}{\sqrt{t}}\frac{6.7\times 10^{-3} m_\eta/GeV}{\sqrt{\ell n(m_\eta^2/m_2^2)-1}} \gtrsim 0.32 \tag{27}$$

In this case, $m_2' > 2$ MeV, beyond the laboratory limit on $m(\nu_\mu)$. This rules out case III and the literal interpretation of the corrections as equivalent to the masses.

Finally, given that $g_{12} \gtrsim 1/19$ from (26) as we have seen, (24a) implies for consistency that $g_{23} \lesssim \frac{1}{36.5} g_{12}$. For an η particle mass in its expected range[17] of 30 GeV, using the cosmological unstable bound

$$m(\nu_\tau) \geq \frac{1}{12}\left(\frac{10^{-10}\mu_B}{\mu_{31}}\right)^2 \tag{28}$$

with

$$\mu_{31} = \frac{e}{32\pi^2}\frac{2g_{23}g_{12}}{m_\eta^2} m_\mu \left(\ell n\frac{m_\eta^2}{m_\mu^2} - 1\right) \tag{29}$$

we can predict the following upper bound on $m(\nu_\tau)$:

$$m(\nu_\tau) \gtrsim 2\ MeV \tag{30}$$

For $m(\nu_\tau)$ at its lower limit, we find $m_{\nu_\tau}/m_\tau = 1.1\times 10^{-3}$, nicely compatible with the see–saw ratios quoted earlier in eq. (16).

8.) Let us summarize our results and the accomplishments of this analysis. We have seen that a model in which the VVO ν_e magnetic moment is satisfied predicts large corrections for the electron neutrino mass, in as least one case. These corrections go in the direction of making the reasonable see–saw a more optimistic predictive tool, as we obtain $m(\nu_e) \sim 20\ eV$, $m(\nu_\mu) \cong 70\ eV - 250\ keV$ and $m(\nu_\tau) \sim 2 - 50\ MeV$ after applying the corrections. In the model, both ν_μ and ν_τ satisfy the unstable cosmological neutrino constraints, while ν_e naturally is consistent with stable bounds. We can have $m(\nu_\mu)$ or the branching ratio of $\mu \to e\gamma$ much smaller than their laboratory bounds and still satisfy the main constraints in the problem. However, both must not be zero. Thus our procedure is consistent with the requirements and has some predictive power.

The analysis above is based on a "poor man's" renormalization evolution, admittedly crude, but appropriate for the spirit of the typical see–saw ratio requirements. The alternative interpretation of one–loop radiative corrections to masses as literal mass matrix elements was also checked. A systematic analysis shows that no region of g_{12}, g_{23}, g_{13} couplings or value of m_η^2 can be found that is consistent with all mass, mixing and VVO moment requirements for the literal interpretation. Since such an interpretation is not really consistent with the formal meaning of the divergent mass corrections, we do not adopt it in any case.

Our basic ideas of renormalizing the bare see–saw to incorporate the effects of running masses can be explored in any renormalizable model. A model with fewer coupling constants is preferable for disentangling the true renormalization group structure.

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Figure Captions

Figure 1: One–loop diagram giving rise to a neutrino magnetic moment.

Figure 2: One–loop contribution to neutrino masses.

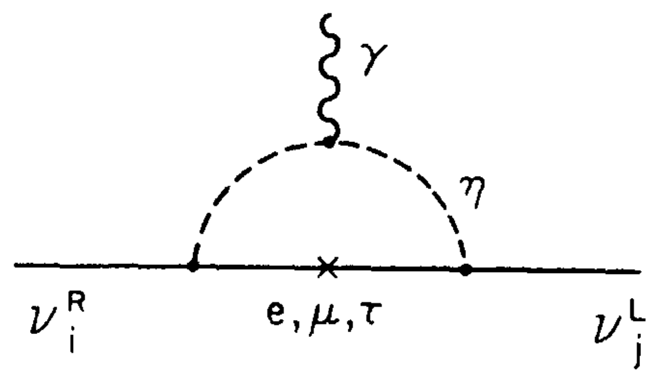

Fig. 1

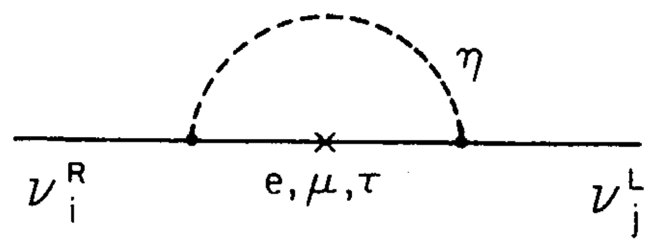

Fig. 2